

Review for the Final Exam

A. Regular Languages

- DFAs, NFAs, ϵ -NFAs You should be able to convert any of the others to a DFA.
- Regular Expressions. It is fairly easy to convert a regular expression to a DFA. It is possible but harder to convert a DFA to a regular expression.
- The Pumping Lemma: If $|w| > p$ then $w=xyz$ where $|xy| \leq p$, y is not empty, and xy^iz is in the language for all $i \geq 0$.
- Properties of Regular Languages: Unions, Intersections, Differences and Complements of regular languages are regular.

B. Context-Free Languages

- Grammars
- PDAs
- To show that grammars generate the same languages as PDAs we found algorithms to convert a grammar to a PDA (easy) and to convert a PDA to a grammar (hard). I won't ask you to do the latter on the final exam.
- Chomsky Normal Form and the algorithm for finding a CNF grammar equivalent to a given grammar.
- The Pumping Lemma for Context-Free languages: If $|z| > p$ then $z=uvwxy$ where $|vwx| \leq p$, v and x aren't both empty, and uv^iwx^iy is in the language for all $i \geq 0$.
- Properties of CF Languages: Unions and concatenations of CF languages are CF. Intersections and Complements of CF languages are not necessarily CF.

C. Turing Machines

- Simple TMs, multi-track, multi-tape and non-deterministic TMs
- Church's Thesis: TMs embody our notion of an algorithm

D. Decidability

- Recursive languages, Recursively enumerable languages, Decidable problems, Recognizable problems
- The diagonal language $\mathcal{L}_d = \{M \mid M \text{ does not accept its own encoding}\}$ is not RE.
- The universal language $\mathcal{L}_u = \{(M,w) \mid M \text{ accepts } w\}$ is RE but not Recursive. The complement of \mathcal{L}_u is not RE.
- The halting language $\mathcal{L}_{\text{halt}} = \{(M,w) \mid M \text{ halts on input } w\}$ is RE but not recursive.
- Rice's Theorem: Any nontrivial property of context-free languages is undecidable.

E. NP-Completeness

- \mathcal{P} is the class of problems that can be solved deterministically in polynomial time
- \mathcal{NP} is the class of problems that can be solved non-deterministically in polynomial time, which usually means that a solution can be verified deterministically in polynomial time.
- A problem is NP-hard if all NP problems reduce to it.
- A problem is NP-Complete if it is both in \mathcal{NP} and NP-hard. If any NP-Complete problem was in \mathcal{P} then \mathcal{P} would equal \mathcal{NP} .
- Cook's (or Cook-Levin) Theorem: SAT is NP-Complete.
- CNF-SAT and 3CNF-SAT are both NP-Complete.
- You should know what all of this means, but I am unlikely to ask you to prove that a specific language is NP-Complete.